

PROBLEM 1 (10 points)

A spin 1 particle, with a gyromagnetic ratio of 3.7, is placed in an external B-field in the u direction such that the Hamiltonian is given by

$$H = \frac{ge}{2mc} B \left(\frac{\sqrt{3}}{2} S_x + \frac{1}{2} S_y \right)$$

Find the energy eigenstates and eigenvalues.

PROBLEM 2 (10 points)

Find the allowed total spin states of two spin 1 particles. Explicitly write out the 9 states which are eigenfunctions of S^2 and S_z . Use notation like $\chi_+\chi_0$ to represent a state where particle 1 has spin $+\hbar$ along the z direction and particle 2 has zero component of spin along the z direction.

PROBLEM 3 (10 points)

The state of an electron is given by $\psi = R(r)[\sqrt{\frac{1}{3}}Y_{10}(\theta, \phi)\chi_+ - \sqrt{\frac{2}{3}}Y_{11}(\theta, \phi)\chi_-]$. Find the possible measured values and the probabilities of the z component of the electron's total angular momentum, (J_z). Do the same for the total angular momentum squared, (J^2).

PROBLEM 4 (10 points)

A harmonic oscillator has a small anharmonic term such that the full Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x^4.$$

Estimate (all) the eigenenergies of system.

PROBLEM 5 (10 points)

If the general form of the spin-orbit coupling for a particle of mass m and spin \vec{S} moving in a potential $V(r)$ is $H_{SO} = \frac{1}{2m^2c^2}\vec{L} \cdot \vec{S} \frac{1}{r} \frac{dV}{dr}$, what is the effect of that coupling on the spectrum of an electron bound in a 3D harmonic oscillator? Give the energy shifts and draw a diagram for the $0s$ and $1p$ states.

PROBLEM 6 (10 points)

We computed that the energies after the fine structure corrections to the hydrogen spectrum are $E_{nlj} = -\frac{\alpha^2 mc^2}{2n^2} + \frac{\alpha^4 mc^2}{8n^4} \left(3 - \frac{4n}{j+\frac{1}{2}}\right)$. Now a weak magnetic field B is applied to hydrogen atoms in the $3d$ state. Calculate the energy shifts of all the $3d$ states (ignoring hyperfine effects). Draw an energy level diagram, showing the quantum numbers of the states and the energy splittings.

PROBLEM 7 (10 points)

Write down the electron configuration for Oxygen ($Z = 8$). Give the quantum numbers of the ground state in spectroscopic notation.

PROBLEM 8 (10 points)

Calculate the excitation energy of the first three rotational levels of the HCl molecule. Give the answer in eV.