1. At $t = 0$, an electron is the spin state

$$\psi(t = 0) = \left( \frac{i}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right).$$

A magnetic field $B$ is applied in the $z$ direction. Find the spin state of the particle as a function of time. Find the expectation value of $S_z$ as a function of time. (10 points)

2. Consider a two dimensional harmonic oscillator problem described by the Hamiltonian

$$H_0 = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2).$$

Accurately calculate the energy shifts to the degenerate first excited states, to first order, if the additional potential $V = \alpha xy$ is applied. Write an expression for the shifted eigenstates. (10 points)

3. A particle like the electron, except that it is spin zero, is bound to a proton in a Hydrogen-like atom. Since the spin is zero, there is no spin-orbit correction and no Darwin term. Calculate the relativistic correction to the energies of the $n = 3$ states assuming the mass of the particle is $m$. The relativistic correction to the Hydrogen Hamiltonian is $H_1 = -\frac{p^4}{8m^4c^2}$. You may use $\langle \psi_{nlm} \mid \frac{1}{r} \mid \psi_{nlm} \rangle = \frac{1}{n^2a_0}$ and $\langle \psi_{nlm} \mid \frac{1}{r^2} \mid \psi_{nlm} \rangle = \frac{1}{n^3a_0^{(l+\frac{1}{2})}}$. (10 points)

4. If the general form of the spin-orbit coupling for a particle of mass $m$ and spin $\vec{S}$ moving in a potential $V(r)$ is $H_{SO} = \frac{1}{2m^2c^2} \vec{L} \cdot \frac{d\vec{S}}{dr}$, what is the effect of that coupling on the spectrum of an electron bound in a 3D harmonic oscillator? Give the energy shifts and and draw a diagram for the 0$p$ and 1$d$ states. (10 points)

5. A hydrogen atom in the ground state is put in a magnetic field. Assume that the energy shift due to the B field is of the same order as the hyperfine splitting of the ground state. Find the eigenenergies of the (four) ground states as a function of the B field strength. Make sure you define any constants (like $A$) you use in terms of fundamental constants. (10 points)

6. The lowest energy excited states of Helium have the electrons in a (1s)(2s) configuration in space and can have total spin 0 or 1. Write out the full (space and spin) states for a) $s = 1$ and $m_s = 0$ and for b) $s = 0$ and $m_s = 0$. Make sure to take account of the Pauli principle in your states. Which state will have the lower energy? (10 points)

7. A Florine ($Z=9$) atom is in its ground state. Write the state in spectroscopic notation. (Ignore the spin of the nucleus.) A weak magnetic field is applied splitting the state. How many energy levels are there now? What is the gyromagnetic ratio for the ground state of Florine. (10 points)
8. Find an expression for the rotational energies of an \( \text{O}_2 \) molecule. Oxygen has \( Z=8 \) and \( A=16 \). What is the energy difference between the lowest and first excited rotational states in eV? (10 points)

9. Assume we have an electron in a standard one dimensional harmonic oscillator of frequency \( \omega \) in its ground state. An weak electric field is applied for a time interval \( T \). Calculate the probability to make a transition to the first excited state. (10 points)

10. Calculate the total decay rate for the \( 3d \rightarrow 2p \) transition in hydrogen. What is the lifetime in nanoseconds? (10 points)