Physics 130B Final Exam   Dec. 8, 2003   Name____________________

1. At $t = 0$, an electron is the spin state
$$\psi(t=0) = \begin{pmatrix} \frac{i}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \end{pmatrix}.$$
A magnetic field B is applied in the z direction. Find the spin state of the particle as a function of time. Find the expectation value of S_x as a function of time. *(10 points)*

2. Consider a two dimensional harmonic oscillator problem described by the Hamiltonian
$$H_0 = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega^2(x^2 + y^2).$$
Accurately calculate the energy shifts to the degenerate first excited states, to first order, if the additional potential $V = \alpha xy$ is applied. Write an expression for the shifted eigenstates. *(10 points)*

3. A particle like the electron, except that it is spin zero, is bound to a proton in a Hydrogen-like atom. Since the spin is zero, there is no spin-orbit correction and no Darwin term. Calculate the relativistic correction to the energies of the $n = 3$ states assuming the mass of the particle is m. The relativistic correction to the Hydrogen Hamiltonian is $H_1 = -\frac{p^4}{8m^3c^2}$. You may use $\langle\psi_{nlm}|\frac{1}{r}|\psi_{nlm}\rangle = \frac{1}{n^2 a_0}$ and $\langle\psi_{nlm}|\frac{1}{r^2}|\psi_{nlm}\rangle = \frac{1}{n^3 a_0^2 (l+\frac{1}{2})}$. *(10 points)*

4. If the general form of the spin-orbit coupling for a particle of mass m and spin $\vec{S}$ moving in a potential $V(r)$ is $H_{SO} = \frac{1}{2m^2c^2}\vec{L}\cdot\vec{S}\frac{1}{r}\frac{dV}{dr}$, what is the effect of that coupling on the spectrum of an electron bound in a 3D harmonic oscillator? Give the energy shifts and and draw a diagram for the $0p$ and $1d$ states. *(10 points)*

5. A hydrogen atom in the ground state is put in a magnetic field. Assume that the energy shift due to the B field is of the same order as the hyperfine splitting of the ground state. Find the eigenenergies of the (four) ground states as a function of the B field strength. Make sure you define any constants (like $\mathcal{A}$) you use in terms of fundamental constants. *(10 points)*

6. The lowest energy excited states of Helium have the electrons in a (1s)(2s) configuration in space and can have total spin 0 or 1. Write out the full (space and spin) states for a) $s = 1$ and $m_s = 0$ and for b) $s = 0$ and $m_s = 0$. Make sure to take account of the Pauli principle in your states. Which state will have the lower energy? *(10 points)*

7. A Florine (Z=9) atom is in its ground state. Write the state in spectroscopic notation. (Ignore the spin of the nucleus.) A weak magnetic field is applied splitting the state. How many energy levels are there now? What is the gyromagnetic ratio for the ground state of Florine. *(10 points)*

8. Find an expression for the rotational energies of an O_2 molecule. Oxygen has Z=8 and A=16. What is the energy difference between the lowest and first excited rotational states in eV? *(10 points)*

9. Assume we have an electron in a standard one dimensional harmonic oscillator of frequency ω in its ground state. An weak electric field is applied for a time interval T. Calculate the probability to make a transition to the first excited state. *(10 points)*

10. Calculate the total decay rate for the $3d \rightarrow 2p$ transition in hydrogen. What is the lifetime in nanoseconds? *(10 points)*