Sample Midterm 1 **May 7, 2012** **Physics 130A**

1. Calculate the **DeBroglie wavelength** for
(a) a proton with 10 MeV kinetic energy,
(b) An electron with 10 MeV kinetic energy, and
(c) a 1 gram lead ball moving with a velocity of 10 cm/sec (one erg is one gram cm^2/sec^2).
Be sure to take account of relativity where needed.
a) $T << mc^2$; $\lambda = \frac{2\pi\hbar}{p}$; $\frac{p^2}{2m} = T$; $p = \sqrt{2mT}$; $\lambda = \frac{2\pi\hbar c}{\sqrt{2mc^2T}} = 9.05F$;
b) $pc = T$; $\lambda = \frac{2\pi\hbar c}{T} = 124F$
c) $\lambda = \frac{2\pi\hbar}{mv} = \frac{2\pi(1.05\times10^{-25}g*cm^2/s)}{10g*cm/s} = 6.6\times10^{-26}$ cm

2. For a free particle, the Hamiltonian operator H is given by $H = p_{op}^2/2m$. Find the functions, $\psi(x)$, which are **eigenfunctions** of both the Hamiltonian and of Parity. Write the eigenfunction that has energy eigenvalue E_0. Now write the corresponding eigenfunctions in momentum space.
$\cos(p_0x/\hbar)$ has positive parity and $\sin(p_0x/\hbar)$ has negative parity with $p_0 = \sqrt{2mE_0}$.
$\frac{1}{\sqrt{2\pi\hbar}}\cos(px/\hbar) = \frac{1}{\sqrt{2\pi\hbar}}\frac{e^{ipx/\hbar}+e^{-ipx/\hbar}}{2}$
The Fourier Transform of $\frac{1}{\sqrt{2\pi\hbar}}e^{ipx/\hbar}$ is $\phi(p) = \delta(p-p_0)$. So the F.T. of the above is $\phi(p) = \frac{\delta(p-p_0)+\delta(p+p_0)}{2}$

3. For a free particle, the total energy operator H is given by $H = p^2/2m$.
a) Compute the **commutators** [H,x] and [H,p].
b) If a particle is in a state of definite energy, what do these commutators tell you about how well we know the particle's position and momentum?
a) $[H,x] = \frac{\hbar p}{im}$ and $[H,p] = 0$
b) Since p commutes with the Hamiltonian, we can know p exactly. Since x does not commute and we are in a definite energy state so that $\Delta E = 0$, we cannot know x at all.

4. Calculate the **commutators** $[p_y, L_x]$ and $[L_y^2, L_z]$.
$[p_y, L_x] = [p_y, yp_z - zp_y] = \frac{\hbar}{i}p_z$
$[L_y^2, L_z] = L_y[L_y, L_z] + [L_y, L_z]L_y = i\hbar(L_yL_x + L_xL_y)$

5. A **beam of particles** of wave-number k (this means e^{ikx}) is incident upon a one dimensional potential $V(x) = \lambda\delta(x)$. Calculate the **probability to be transmitted**. Graph it as a function of k.
For $x < 0$ $e^{ikx} + Re^{-ikx}$; for $x > 0$, Te^{ikx}; Wavefunction is continuous $1 + R = T$. Discontinuity in first derivative $ikT - (ik - ikR) = \frac{2m\lambda}{\hbar^2}T$; $T = \frac{ik}{ik - m\lambda/\hbar^2}$; $P_T = |T|^2 = \frac{k^2}{k^2+m^2\lambda^2/\hbar^4}$. Show a graph.

6. A 1D harmonic oscillator is in a **linear combination of the energy eigenstates** $\psi(t=0) = \sqrt{\frac{1}{4}}u_0 - i\sqrt{\frac{3}{4}}u_1$. Find the expected value of p at a later time t.
$\psi(t) = \sqrt{\frac{1}{4}}u_0e^{-i\omega t/2} - i\sqrt{\frac{3}{4}}u_1e^{-3i\omega t/2}$; $p = i\sqrt{\frac{m\hbar\omega}{2}}(A^\dagger - A)$; $\langle p\rangle = i\sqrt{\frac{m\hbar\omega}{2}}\langle\sqrt{\frac{1}{4}}u_0e - i\sqrt{\frac{3}{4}}u_1e^{-i\omega t}|(A^\dagger - A)|\sqrt{\frac{1}{4}}u_0e - i\sqrt{\frac{3}{4}}u_1e^{-i\omega t}\rangle$ $\langle p\rangle = i\sqrt{\frac{m\hbar\omega}{2}}\sqrt{\frac{3}{16}}(ie^{-i\omega t} + ie^{i\omega t}) = -\sqrt{\frac{3m\hbar\omega}{32}}2\cos(\omega t) = -\sqrt{\frac{3m\hbar\omega}{8}}\cos(\omega t)$