1. Calculate the DeBroglie wavelength for
   (a) a proton with 10 MeV kinetic energy,
   (b) An electron with 10 MeV kinetic energy, and
   (c) a 1 gram lead ball moving with a velocity of 10 cm/sec (one erg is one gram cm²/sec²).
   Be sure to take account of relativity where needed.
   
   a) \( T << mc^2; \lambda = \frac{2\pi\hbar}{p}; \frac{p^2}{2m} = T; p = \sqrt{2mT}; \lambda = \frac{2\pi\hbar}{\sqrt{2mc^2T}} = 9.05F; \)
   
   b) \( pc = T; \lambda = \frac{2\pi\hbar}{p} = 124F \)
   
   c) \( \lambda = \frac{2\pi\hbar}{mv} = \frac{2\pi(1.05\times10^{-25}g\times cm^2/s)}{10g\times cm/s} = 6.6\times10^{-26} \text{ cm} \)

2. For a free particle, the Hamiltonian operator \( H \) is given by \( H = p^2/2m \). Find the functions, \( \psi(x) \), which are eigenfunctions of both the Hamiltonian and of Parity. Write the eigenfunction that has energy eigenvalue \( E_0 \). Now write the corresponding eigenfunctions in momentum space.

   \[ \cos(p_0x/\hbar) \] has positive parity and \( \sin(p_0x/\hbar) \) has negative parity with \( p_0 = \sqrt{2mE_0} \).

   \[ \frac{1}{\sqrt{2\pi\hbar}} \cos(px/\hbar) = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} + e^{-ipx/\hbar} \]

   The Fourier Transform of \( \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \) is \( \phi(p) = \delta(p - p_0) \). So the F.T. of the above is \( \phi(p) = \delta(p-p_0)+\delta(p+p_0)/2 \)

3. For a free particle, the total energy operator \( H \) is given by \( H = p^2/2m \).
   
   a) Compute the commutators \([H,x]\) and \([H,p]\).
   
   b) If a particle is in a state of definite energy, what do these commutators tell you about how well we know the particle’s position and momentum?
   
   a) \([H, x] = \frac{\hbar p}{im} \) and \([H, p] = 0 \)
   
   b) Since \( p \) commutes with the Hamiltonian, we can know \( p \) exactly. Since \( x \) does not commute and we are in a definite energy state so that \( \Delta E = 0 \), we cannot know \( x \) at all.

4. Calculate the commutators \([p_y, L_x]\) and \([L_y^2, L_z]\).

   \[ [p_y, L_x] = [p_y, yp_z - zp_y] = \frac{\hbar}{i} p_z \]

   \[ [L_y^2, L_z] = L_y[L_y, L_z] + [L_y, L_z]L_y = i\hbar(L_yL_z + L_zL_y) \]

5. A beam of particles of wave-number \( k \) (this means \( e^{ikx} \)) is incident upon a one dimensional potential \( V(x) = \lambda\delta(x) \). Calculate the probability to be transmitted. Graph it as a function of \( k \).

   For \( x < 0 \) \( e^{ikx} + Re^{-ikx} \); for \( x > 0 \), \( Te^{ikx} \); Wavefunction is continuous \( 1 + R = T \). Discontinuity in first derivative \( ikT - (ik - ikR) = \frac{2m\lambda T}{\hbar}; T = \frac{ik}{ik - m\lambda/\hbar}; P_T = |T|^2 = \frac{k^2}{k^2 + m^2\lambda^2/\hbar^2} \). Show a graph.

6. A 1D harmonic oscillator is in a linear combination of the energy eigenstates \( \psi(t = 0) = \sqrt{\frac{1}{2}} u_0 - i\sqrt{\frac{1}{4}} u_1 \). Find the expected value of \( p \) at a later time \( t \).

   \( \psi(t) = \sqrt{T} u_0 e^{-i\omega t / 2} - i\sqrt{\frac{1}{4}} u_1 e^{-3i\omega t / 2} \); \( p = i\sqrt{\frac{m\hbar}{2}} (A^\dagger - A) \); \( \langle p \rangle = i\sqrt{\frac{m\hbar}{2}} \sqrt{T} u_0 e^{-i\sqrt{\frac{1}{4}} u_1 e^{-i\omega t}} \langle A^\dagger - A \rangle \sqrt{T} u_0 e^{-i\sqrt{\frac{1}{4}} u_1 e^{-i\omega t}} \) \( \langle p \rangle = \sqrt{\frac{m\hbar}{2}} \sqrt{T} u_0 e^{-i\omega t} + i e^{i\omega t} \) \( \langle p \rangle = -\sqrt{\frac{3m\hbar}{32}} 2 \cos(\omega t) = -\sqrt{\frac{3m\hbar}{8}} \cos(\omega t) \)