Midterm 2 **May 31, 2006** **Physics 130A**

NAME	Secret Number	Score

Constants

$\hbar = 1.05 \times 10^{-27}$ erg sec	$c = 3.00 \times 10^{10}$ cm/sec	$e = 1.602 \times 10^{-19}$ coulomb
$1eV = 1.602 \times 10^{-12}$ erg	$\alpha = \frac{e^2}{\hbar c} = 1/137$	$\hbar c = 1973$ eV Å$= 197.3$ MeV F
1 Å$= 1.0 \times 10^{-8}$ cm	1 Fermi $= 1.0 \times 10^{-13}$ cm	$a_0 = \frac{\hbar}{\alpha m_e c} = 0.529 \times 10^{-8}$ cm
$m_p = 938.3$ MeV$/c^2$	$m_n = 939.6$ MeV$/c^2$	$m_e = 9.11 \times 10^{-28}$ g $= 0.511$ MeV$/c^2$
$k_B = 1.38 \times 10^{-16}$ erg/°K	$g_e = 2 + \frac{\alpha}{\pi}$	$g_p = 5.6$
$\mu_{Bohr} = \frac{e\hbar}{2m_e c} = 0.579 \times 10^{-8}$ eV/gauss		$\int\limits_{-\infty}^{\infty} dx\, f(x)\, \delta(g(x)) = \left[\frac{1}{\lvert\frac{dg}{dx}\rvert} f(x)\right]_{g(x)=0}$
$\int\limits_{-\infty}^{\infty} dx\, f(x)\, \delta(x-a) = f(a)$	$\int\limits_{-\infty}^{\infty} dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$	use $\frac{\partial}{\partial a}$ for other forms
$e^A = \sum\limits_{n=0}^{\infty} \frac{A^n}{n!}$	$\sin\theta = \sum\limits_{n=1,3,5...}^{\infty} \frac{\theta^n}{n!}(-1)^{\frac{n-1}{2}}$	$\cos\theta = \sum\limits_{n=0,2,4...}^{\infty} \frac{\theta^n}{n!}(-1)^{\frac{n}{2}}$
$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$	$\int\limits_{0}^{\infty} dr\, r^n e^{-ar} = \frac{n!}{a^{n+1}}$	$E = \sqrt{m^2c^4 + p^2c^2}$

HARMONIC OSCILLATOR

$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \hbar\omega A^\dagger A + \frac{1}{2}\hbar\omega$		$E_n = (n+\frac{1}{2})\hbar\omega \quad n = 0, 1, 2...$
$u_n(x) = \sum\limits_{k=0}^{\infty} a_k y^k e^{-y^2/2}$	$a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k$	$y = \sqrt{\frac{m\omega}{\hbar}}x$
$A = (\sqrt{\frac{m\omega}{2\hbar}}x + i\frac{p}{\sqrt{2m\hbar\omega}})$	$A^\dagger = (\sqrt{\frac{m\omega}{2\hbar}}x - i\frac{p}{\sqrt{2m\hbar\omega}})$	$[A, A^\dagger] = 1$
$A^\dagger \lvert n\rangle = \sqrt{(n+1)}\, \lvert n+1\rangle$	$A \lvert n\rangle = \sqrt{(n)}\, \lvert n-1\rangle$	$u_0(x) = (\frac{m\omega}{\hbar\pi})^{\frac{1}{4}}\, e^{-m\omega x^2/2\hbar}$

ANGULAR MOMENTUM

$[L_i, L_j] = i\hbar\epsilon_{ijk}L_k$	$[L^2, L_i] = 0$	$\int Y^*_{\ell m} Y_{\ell' m'} d\Omega = \delta\ell\ell'\delta_{mm'}$
$L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$	$L_z Y_{\ell m} = m\hbar Y_{\ell m}$	$-\ell \le m \le \ell$
$L_\pm = L_x \pm iL_y$	$L_\pm Y_{\ell m} = \hbar\sqrt{\ell(\ell+1) - m(m\pm1)}\; Y_{\ell, m\pm1}$	
$Y_{00} = \frac{1}{\sqrt{4\pi}}$	$Y_{11} = -\sqrt{\frac{3}{8\pi}}\, e^{i\phi}\, \sin\theta$	$Y_{10} = \sqrt{\frac{3}{4\pi}}\, \cos\theta$
$Y_{22} = \sqrt{\frac{15}{32\pi}}\, e^{2i\phi}\, \sin^2\theta$	$Y_{21} = -\sqrt{\frac{15}{8\pi}}\, e^{i\phi}\, \sin\theta\cos\theta$	$Y_{20} = \sqrt{\frac{5}{16\pi}}\, (3\cos^2\theta - 1)$
$Y_{\ell\ell} = e^{i\ell\phi}\, \sin^\ell\theta$	$Y_{\ell(-m)} = (-1)^m Y^*_{\ell m}$	$Y_{\ell m}(\pi-\theta, \phi+\pi) = (-1)^\ell\, Y_{\ell m}(\theta, \phi)$
$\frac{-\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right] R_{n\ell}(r) + \left(V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right) R_{n\ell}(r) = ER_{n\ell}(r)$		

GENERAL WAVE MECHANICS

$E = h\nu = \hbar\omega$	$\lambda = h/p$	$p = \hbar k$
$\Delta p \, \Delta x \geq \frac{\hbar}{2}$	$\Delta A \, \Delta B \geq \langle \frac{1}{2}[A, B] \rangle$	$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$
$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int\limits_{-\infty}^{\infty} dp \, \phi(p) \, e^{ipx/\hbar}$		$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int\limits_{-\infty}^{\infty} dx \, \psi(x) \, e^{-ipx/\hbar}$
$p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x}$	$E_{op} = i\hbar \frac{\partial}{\partial t}$	$x_{op} = i\hbar \frac{\partial}{\partial p}$
$H u_j(x) = E_j u_j(x)$	$\psi_j(x, t) = u_j(x) e^{-iE_j t/\hbar}$	$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar \frac{\partial \psi}{\partial t}$
$\psi(x)$ continuous	$\frac{d\psi}{dx}$ continous if V finite	
$\Delta \frac{d\psi}{dx} = \frac{2m\lambda}{\hbar^2} \psi(a)$ for $V(x) = \lambda \delta(x - a)$		
$\langle \phi \| \psi \rangle = \int\limits_{-\infty}^{\infty} dx \phi^*(x) \psi(x)$	$\langle u_i \| u_j \rangle = \delta_{ij}$	$\sum\limits_i \| u_i \rangle \langle u_i \| = 1$
$\phi = \sum\limits_i a_i u_i$	$a_i = \langle u_i \| \phi \rangle$	$\psi(x) = \langle x \| \psi \rangle$
$\langle \phi \| A \| \psi \rangle = \langle \psi \| A \| \phi \rangle^* = \langle \phi \| A \psi \rangle = \langle A^\dagger \phi \| \psi \rangle$		$\phi(p) = \langle p \| \psi \rangle$
		$H\psi = E\psi$
$[p_x, x] = \frac{\hbar}{i}$	$[L_x, L_y] = i\hbar L_z$	$[L^2, L_z] = 0$
$\psi_i = \langle u_i \| \psi \rangle$	$A_{ij} = \langle u_i \| A \| u_j \rangle$	$\frac{d\langle A \rangle}{dt} = \langle \frac{\partial A}{\partial t} \rangle + \frac{i}{\hbar} \langle [H, A] \rangle$

HYDROGEN ATOM

$H = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$	$\psi_{n\ell m} = R_{n\ell}(r) Y_{\ell m}(\theta, \phi)$	$E_n = -\frac{Z^2 \alpha^2 \mu c^2}{2n^2} = -\frac{13.6}{n^2}$ eV
$n = n_r + \ell + 1$	$a_0 = \frac{\hbar}{\alpha \mu c}$	$\ell = 0, 1, ..., n - 1$
$R_{n\ell}(\rho) = \rho^\ell \sum\limits_{k=0}^{\infty} a_k \rho^k e^{-\rho/2}$	$a_{k+1} = \frac{k+\ell+1-n}{(k+1)(k+2\ell+2)} a_k$	$\rho = \sqrt{\frac{-8\mu E}{\hbar^2}} r = \frac{2r}{na_0}$
$R_{10} = 2(\frac{Z}{a_0})^{\frac{3}{2}} \, e^{\frac{-Zr}{a_0}}$	$R_{20} = 2(\frac{Z}{2a_0})^{\frac{3}{2}} (1 - \frac{Zr}{2a_0}) \, e^{\frac{-Zr}{2a_0}}$	$R_{21} = \frac{1}{\sqrt{3}} (\frac{Z}{2a_0})^{\frac{3}{2}} (\frac{Zr}{a_0}) \, e^{\frac{-Zr}{2a_0}}$
$R_{n,n-1} \propto r^{n-1} \, e^{-Zr/na_0}$	$\mu = \frac{m_1 m_2}{m_1 + m_2}$	$\langle \psi_{n\ell m} \| \frac{e^2}{r} \| \psi_{n\ell m} \rangle = \frac{Ze^2}{n^2 a_0} = \frac{Z\alpha^2 \mu c^2}{n^2}$

1. At $t = 0$, a one dimensional harmonic oscillator is in the state $\psi(t=0) = \sqrt{\frac{2}{3}}u_1 - i\sqrt{\frac{1}{3}}u_2$. Calculate the expected value of p as a function of time. If a measurement of the energy is made, what are the possible outcomes and what are the corresponding probabilities?

2. Assume the potential for a particle of mass m is independent of y, $V(\vec{r}) = V(x, z)$. What symmetry is the problem said to have? Use an infinitesimal step in the symmetric direction to show that some operator commutes with the Hamiltonian. What quantity is conserved (that is does not change with time) in this potential?

3. Use the commutator relation between the x, y, and z components of angular momentum to derive $[L_+, L^2]$ and $[L_+, L_z]$. Now use these commutators to show that L_+ is the raising operator for the harmonic oscillator energy. Finally, compute the constant in the equation $L_+ Y_{\ell m} = C\ Y_{\ell(m+1)}$.