Midterm 1 **May 2, 2005** **Physics 130A**

NAME	Secret Number	Score

(write your name and secret number on the blue book)

1. What is the maximum wavelength of electromagnetic radiation which can eject electrons from a metal having a work function of 1.97 eV? *(5 points)*

2. a) What is the deBroglie wavelength of an electron with 5 eV of kinetic energy?

   b) What is the deBroglie wavelength of an electron with 100 MeV of kinetic energy?

   (6 points)

3. Given the following one dimensional probability amplitudes in the position variable x, compute the probability distribution in momentum space. Show that the uncertainty principle is roughly satisfied.

   a) $\psi(x) = \frac{1}{\sqrt{2a}}$ for $-a < x < a$, otherwise $\psi(x) = 0$.

   b) $\psi(x) = \delta(x - x_0)$

   (10 points)

4. A particle is confined to a box of length L in one dimension. It is initially in the ground state. Suddenly, one wall of the box is moved outward making a new box of length $2L$. What is the probability that the particle is in the ground state of the new box? You may find it useful to know that $\int \sin(Ax)\sin(Bx)dx = \frac{\sin((A-B)x)}{2(A-B)} - \frac{\sin((A+B)x)}{2(A+B)}$ *(7 points)*

Constants

$\hbar = 1.05 \times 10^{-27}$ erg sec	$c = 3.00 \times 10^{10}$ cm/sec	$e = 1.602 \times 10^{-19}$ coulomb
$1eV = 1.602 \times 10^{-12}$ erg	$\alpha = \frac{e^2}{\hbar c} = 1/137$	$\hbar c = 1973$ eV Å$= 197.3$ MeV F
1 Å$= 1.0 \times 10^{-8}$ cm	1 Fermi $= 1.0 \times 10^{-13}$ cm	$a_0 = \frac{\hbar}{\alpha m_e c} = 0.529 \times 10^{-8}$ cm
$m_p = 938.3$ MeV$/c^2$	$m_n = 939.6$ MeV$/c^2$	$m_e = 9.11 \times 10^{-28}$ g $= 0.511$ MeV$/c^2$
$k_B = 1.38 \times 10^{-16}$ erg/°K	$g_e = 2 + \frac{\alpha}{\pi}$	$g_p = 5.6$
$\mu_{Bohr} = \frac{e\hbar}{2m_e c} = 0.579 \times 10^{-8}$ eV/gauss		$\int\limits_{-\infty}^{\infty} dx\ f(x)\ \delta(g(x)) = \left[\frac{1}{\lvert\frac{dg}{dx}\rvert} f(x)\right]_{g(x)=0}$
$\int\limits_{-\infty}^{\infty} dx\ f(x)\ \delta(x-a) = f(a)$	$\int\limits_{-\infty}^{\infty} dx\ e^{-ax^2} = \sqrt{\frac{\pi}{a}}$	use $\frac{\partial}{\partial a}$ for other forms
$e^A = \sum\limits_{n=0}^{\infty} \frac{A^n}{n!}$	$\sin\theta = \sum\limits_{n=1,3,5...}^{\infty} \frac{\theta^n}{n!}(-1)^{\frac{n-1}{2}}$	$\cos\theta = \sum\limits_{n=0,2,4...}^{\infty} \frac{\theta^n}{n!}(-1)^{\frac{n}{2}}$
$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$	$\int\limits_{0}^{\infty} dr\ r^n e^{-ar} = \frac{n!}{a^{n+1}}$	$E = \sqrt{m^2c^4 + p^2c^2}$

HARMONIC OSCILLATOR

$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \hbar\omega A^\dagger A + \frac{1}{2}\hbar\omega$		$E_n = (n + \frac{1}{2})\hbar\omega \quad n = 0, 1, 2...$
$u_n(x) = \sum\limits_{k=0}^{\infty} a_k y^k e^{-y^2/2}$	$a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k$	$y = \sqrt{\frac{m\omega}{\hbar}} x$
$A = (\sqrt{\frac{m\omega}{2\hbar}} x + i\frac{p}{\sqrt{2m\hbar\omega}})$	$A^\dagger = (\sqrt{\frac{m\omega}{2\hbar}} x - i\frac{p}{\sqrt{2m\hbar\omega}})$	$[A, A^\dagger] = 1$
$A^\dagger \lvert n\rangle = \sqrt{(n+1)}\ \lvert n+1\rangle$	$A \lvert n\rangle = \sqrt{(n)}\ \lvert n-1\rangle$	$u_0(x) = (\frac{m\omega}{\hbar\pi})^{\frac{1}{4}}\ e^{-m\omega x^2/2\hbar}$

ANGULAR MOMENTUM

$[L_i, L_j] = i\hbar\epsilon_{ijk} L_k$	$[L^2, L_i] = 0$	$\int Y^*_{\ell m} Y_{\ell' m'} d\Omega = \delta\ell\ell' \delta_{mm'}$
$L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$	$L_z Y_{\ell m} = m\hbar Y_{\ell m}$	$-\ell \leq m \leq \ell$
$L_\pm = L_x \pm iL_y$	$L_\pm Y_{\ell m} = \hbar\sqrt{\ell(\ell+1) - m(m\pm 1)}\ Y_{\ell, m\pm 1}$	
$Y_{00} = \frac{1}{\sqrt{4\pi}}$	$Y_{11} = -\sqrt{\frac{3}{8\pi}}\ e^{i\phi}\ \sin\theta$	$Y_{10} = \sqrt{\frac{3}{4\pi}}\ \cos\theta$
$Y_{22} = \sqrt{\frac{15}{32\pi}}\ e^{2i\phi}\ \sin^2\theta$	$Y_{21} = -\sqrt{\frac{15}{8\pi}}\ e^{i\phi}\ \sin\theta\cos\theta$	$Y_{20} = \sqrt{\frac{5}{16\pi}}\ (3\cos^2\theta - 1)$
$Y_{\ell\ell} = e^{i\ell\phi}\ \sin^\ell\theta$	$Y_{\ell(-m)} = (-1)^m Y^*_{\ell m}$	$Y_{\ell m}(\pi - \theta, \phi + \pi) = (-1)^\ell\ Y_{\ell m}(\theta, \phi)$
$\frac{-\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right] R_{n\ell}(r) + \left(V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right) R_{n\ell}(r) = E R_{n\ell}(r)$		

GENERAL WAVE MECHANICS

$E = h\nu = \hbar\omega$	$\lambda = h/p$	$p = \hbar k$
$\Delta p \, \Delta x \geq \frac{\hbar}{2}$	$\Delta A \, \Delta B \geq \langle \frac{1}{2}[A, B] \rangle$	$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$
$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int\limits_{-\infty}^{\infty} dp \; \phi(p) \; e^{ipx/\hbar}$		$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int\limits_{-\infty}^{\infty} dx \; \psi(x) \; e^{-ipx/\hbar}$
$p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x}$	$E_{op} = i\hbar \frac{\partial}{\partial t}$	$x_{op} = i\hbar \frac{\partial}{\partial p}$
$H u_j(x) = E_j u_j(x)$	$\psi_j(x, t) = u_j(x) e^{-iE_j t/\hbar}$	$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar \frac{\partial \psi}{\partial t}$
$\psi(x)$ continuous	$\frac{d\psi}{dx}$ continous if V finite	
$\Delta \frac{d\psi}{dx} = \frac{2m\lambda}{\hbar^2} \psi(a)$ for $V(x) = \lambda \delta(x - a)$		
$\langle \phi \vert \psi \rangle = \int\limits_{-\infty}^{\infty} dx \phi^*(x) \psi(x)$	$\langle u_i \vert u_j \rangle = \delta_{ij}$	$\sum\limits_i \vert u_i \rangle \langle u_i \vert = 1$
$\phi = \sum\limits_i a_i u_i$	$a_i = \langle u_i \vert \phi \rangle$	$\psi(x) = \langle x \vert \psi \rangle$
$\langle \phi \vert A \vert \psi \rangle = \langle \psi \vert A \vert \phi \rangle^* = \langle \phi \vert A \psi \rangle = \langle A^\dagger \phi \vert \psi \rangle$		$\phi(p) = \langle p \vert \psi \rangle$
		$H\psi = E\psi$
$[p_x, x] = \frac{\hbar}{i}$	$[L_x, L_y] = i\hbar L_z$	$[L^2, L_z] = 0$
$\psi_i = \langle u_i \vert \psi \rangle$	$A_{ij} = \langle u_i \vert A \vert u_j \rangle$	$\frac{d\langle A \rangle}{dt} = \langle \frac{\partial A}{\partial t} \rangle + \frac{i}{\hbar} \langle [H, A] \rangle$

HYDROGEN ATOM

$H = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$	$\psi_{n\ell m} = R_{n\ell}(r) Y_{\ell m}(\theta, \phi)$	$E_n = -\frac{Z^2 \alpha^2 \mu c^2}{2n^2} = -\frac{13.6}{n^2}$ eV
$n = n_r + \ell + 1$	$a_0 = \frac{\hbar}{\alpha \mu c}$	$\ell = 0, 1, ..., n - 1$
$R_{n\ell}(\rho) = \rho^\ell \sum\limits_{k=0}^{\infty} a_k \rho^k e^{-\rho/2}$	$a_{k+1} = \frac{k+\ell+1-n}{(k+1)(k+2\ell+2)} a_k$	$\rho = \sqrt{\frac{-8\mu E}{\hbar^2}} r = \frac{2r}{na_0}$
$R_{10} = 2(\frac{Z}{a_0})^{\frac{3}{2}} \; e^{\frac{-Zr}{a_0}}$	$R_{20} = 2(\frac{Z}{2a_0})^{\frac{3}{2}}(1 - \frac{Zr}{2a_0}) \; e^{\frac{-Zr}{2a_0}}$	$R_{21} = \frac{1}{\sqrt{3}}(\frac{Z}{2a_0})^{\frac{3}{2}}(\frac{Zr}{a_0}) \; e^{\frac{-Zr}{2a_0}}$
$R_{n,n-1} \; \propto \; r^{n-1} \; e^{-Zr/na_0}$	$\mu = \frac{m_1 m_2}{m_1 + m_2}$	$\langle \psi_{n\ell m} \vert \frac{e^2}{r} \vert \psi_{n\ell m} \rangle = \frac{Ze^2}{n^2 a_0} = \frac{Z\alpha^2 \mu c^2}{n^2}$