**FINAL** **June 14, 2001** **Physics 130A**

| **NAME** | **Secret Number** | **Score** |
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| $\hbar = 1.05 \times 10^{-27}$ erg sec | $c = 3.00 \times 10^{10}$ cm/sec | $e = 1.602 \times 10^{-19}$ coulomb |
| $1eV = 1.602 \times 10^{-12}$ erg | $\alpha = \frac{e^2}{\hbar c} = 1/137$ | $\hbar c = 1973$ eV Å$= 197.3$ MeV F |
| 1 Å$= 1.0 \times 10^{-8}$ cm | 1 Fermi $= 1.0 \times 10^{-13}$ cm | $a_0 = \frac{\hbar}{\alpha m_e c} = 0.529 \times 10^{-8}$ cm |
| $m_p = 938.3$ MeV/$c^2$ | $m_n = 939.6$ MeV/$c^2$ | $m_e = 9.11 \times 10^{-28}$ g $= 0.511$ MeV/$c^2$ |
| $k_B = 1.38 \times 10^{-16}$ erg/°K | $g_e = 2 + \frac{\alpha}{\pi}$ | $g_p = 5.6$ |
| $\mu_{Bohr} = \frac{e\hbar}{2m_e c} = 0.579 \times 10^{-8}$ eV/gauss | | $\int\limits_{-\infty}^{\infty} dx\ f(x)\ \delta(g(x)) = \left[\frac{1}{\lvert\frac{dg}{dx}\rvert} f(x)\right]_{g(x)=0}$ |
| $\int\limits_{-\infty}^{\infty} dx\ f(x)\ \delta(x-a) = f(a)$ | $\int\limits_{-\infty}^{\infty} dx\ e^{-ax^2} = \sqrt{\frac{\pi}{a}}$ | use $\frac{\partial}{\partial a}$ for other forms |
| $e^A = \sum\limits_{n=0}^{\infty} \frac{A^n}{n!}$ | $\sin\theta = \sum\limits_{n=1,3,5...}^{\infty} \frac{\theta^n}{n!}(-1)^{\frac{n-1}{2}}$ | $\cos\theta = \sum\limits_{n=0,2,4...}^{\infty} \frac{\theta^n}{n!}(-1)^{\frac{n}{2}}$ |
| $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$ | $\int\limits_{0}^{\infty} dr\ r^n e^{-ar} = \frac{n!}{a^{n+1}}$ | $E = \sqrt{m^2c^4 + p^2c^2}$ |

## GENERAL WAVE MECHANICS

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| $E = h\nu = \hbar\omega$ | $\lambda = h/p$ | $p = \hbar k$ |
| $\Delta p\ \Delta x \geq \frac{\hbar}{2}$ | $\Delta A\ \Delta B \geq \langle \frac{1}{2}[A,B] \rangle$ | $\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$ |
| $\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int\limits_{-\infty}^{\infty} dp\ \phi(p)\ e^{ipx/\hbar}$ | | $\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int\limits_{-\infty}^{\infty} dx\ \psi(x)\ e^{-ipx/\hbar}$ |
| $p_{op} = \frac{\hbar}{i} \frac{\partial}{\partial x}$ | $E_{op} = i\hbar \frac{\partial}{\partial t}$ | $x_{op} = i\hbar \frac{\partial}{\partial p}$ |
| $H u_j(x) = E_j u_j(x)$ | $\psi_j(x,t) = u_j(x) e^{-iE_j t/\hbar}$ | $\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar \frac{\partial \psi}{\partial t}$ |
| $\psi(x)$ continuous | $\frac{d\psi}{dx}$ continous if $V$ finite | |
| $\Delta \frac{d\psi}{dx} = \frac{2m\lambda}{\hbar^2} \psi(a)$ for $V(x) = \lambda\delta(x-a)$ | | |
| $\langle \phi \vert \psi \rangle = \int\limits_{-\infty}^{\infty} dx \phi^*(x)\psi(x)$ | $\langle u_i \vert u_j \rangle = \delta_{ij}$ | $\sum\limits_i \vert u_i \rangle \langle u_i \vert = 1$ |
| $\phi = \sum\limits_i a_i u_i$ | $a_i = \langle u_i \vert \phi \rangle$ | $\psi(x) = \langle x \vert \psi \rangle$ |
| $\langle \phi \vert A \vert \psi \rangle = \langle \psi \vert A \vert \phi \rangle^* = \langle \phi \vert A\psi \rangle = \langle A^\dagger \phi \vert \psi \rangle$ | | $\phi(p) = \langle p \vert \psi \rangle$ |
| | | $H\psi = E\psi$ |
| $[p_x, x] = \frac{\hbar}{i}$ | $[L_x, L_y] = i\hbar L_z$ | $[L^2, L_z] = 0$ |
| $\psi_i = \langle u_i \vert \psi \rangle$ | $A_{ij} = \langle u_i \vert A \vert u_j \rangle$ | $\frac{d\langle A \rangle}{dt} = \langle \frac{\partial A}{\partial t} \rangle + \frac{i}{\hbar} \langle [H,A] \rangle$ |

## HARMONIC OSCILLATOR

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| $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \hbar\omega A^\dagger A + \frac{1}{2}\hbar\omega$ | | $E_n = (n+\frac{1}{2})\hbar\omega \quad n = 0, 1, 2...$ |
| $u_n(x) = \sum_{k=0}^{\infty} a_k y^k e^{-y^2/2}$ | $a_{k+2} = \frac{2(k-n)}{(k+1)(k+2)} a_k$ | $y = \sqrt{\frac{m\omega}{\hbar}}x$ |
| $A = (\sqrt{\frac{m\omega}{2\hbar}}x + i\frac{p}{\sqrt{2m\hbar\omega}})$ | $A^\dagger = (\sqrt{\frac{m\omega}{2\hbar}}x - i\frac{p}{\sqrt{2m\hbar\omega}})$ | $[A, A^\dagger] = 1$ |
| $A^\dagger \|n\rangle = \sqrt{(n+1)}\ \|n+1\rangle$ | $A \|n\rangle = \sqrt{(n)}\ \|n-1\rangle$ | $u_0(x) = (\frac{m\omega}{\hbar\pi})^{\frac{1}{4}}\ e^{-m\omega x^2/2\hbar}$ |

## ANGULAR MOMENTUM

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| $[L_i, L_j] = i\hbar\epsilon_{ijk}L_k$ | $[L^2, L_i] = 0$ | |
| $L^2 Y_{\ell m} = \ell(\ell+1)\hbar^2 Y_{\ell m}$ | $L_z Y_{\ell m} = m\hbar Y_{\ell m}$ | $-\ell \le m \le \ell$ |
| $L_\pm = L_x \pm iL_y$ | $L_\pm Y_{\ell m} = \hbar\sqrt{\ell(\ell+1) - m(m\pm 1)}\ Y_{\ell, m\pm 1}$ | |
| $Y_{00} = \frac{1}{\sqrt{4\pi}}$ | $Y_{11} = -\sqrt{\frac{3}{8\pi}}\ e^{i\phi}\ \sin\theta$ | $Y_{10} = \sqrt{\frac{3}{4\pi}}\ \cos\theta$ |
| $Y_{22} = \sqrt{\frac{15}{32\pi}}\ e^{2i\phi}\ \sin^2\theta$ | $Y_{21} = -\sqrt{\frac{15}{8\pi}}\ e^{i\phi}\ \sin\theta\cos\theta$ | $Y_{20} = \sqrt{\frac{5}{16\pi}}\ (3\cos^2\theta - 1)$ |
| $Y_{\ell\ell} = e^{i\ell\phi}\ \sin^\ell\theta$ | $Y_{\ell(-m)} = (-1)^\ell Y^*_{\ell m}$ | $Y_{\ell m}(\pi-\theta, \phi+\pi) = (-1)^\ell\ Y_{\ell m}(\theta, \phi)$ |
| $\frac{-\hbar^2}{2\mu}\left[\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right] R_{n\ell}(r) + \left(V(r) + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}\right) R_{n\ell}(r) = E R_{n\ell}(r)$ | | |
| $j_0(kr) = \frac{\sin(kr)}{kr}$ | $n_0(kr) = -\frac{\cos(kr)}{kr}$ | $h^{(1)}_\ell(kr) = j_\ell(kr) + in_\ell(kr)$ |
| $H = H_0 - \vec{\mu}\cdot\vec{B}$ | $\vec{\mu} = \frac{e}{2mc}\vec{L}$ | $\vec{\mu} = \frac{ge}{2mc}\vec{S}$ |
| $S_i = \frac{\hbar}{2}\sigma_i$ | | |
| $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ | $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ | $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ |

## HYDROGEN ATOM

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| $H = \frac{p^2}{2\mu} - \frac{Ze^2}{r}$ | $\psi_{n\ell m} = R_{n\ell}(r)Y_{\ell m}(\theta, \phi)$ | $E_n = -\frac{Z^2\alpha^2\mu c^2}{2n^2} = -\frac{13.6}{n^2}$ eV |
| $n = n_r + \ell + 1$ | $a_0 = \frac{\hbar}{\alpha\mu c}$ | $\ell = 0, 1, ..., n-1$ |
| $R_{n\ell}(\rho) = \rho^\ell \sum_{k=0}^{\infty} a_k\rho^k e^{-\rho/2}$ | $a_{k+1} = \frac{k+\ell+1-n}{(k+1)(k+2\ell+2)} a_k$ | $\rho = \sqrt{\frac{-8\mu E}{\hbar^2}}r = \frac{2r}{na_0}$ |
| $R_{10} = 2(\frac{Z}{a_0})^{\frac{3}{2}}\ e^{\frac{-Zr}{a_0}}$ | $R_{20} = 2(\frac{Z}{2a_0})^{\frac{3}{2}}(1 - \frac{Zr}{2a_0})\ e^{\frac{-Zr}{2a_0}}$ | $R_{21} = \frac{1}{\sqrt{3}}(\frac{Z}{2a_0})^{\frac{3}{2}}(\frac{Zr}{a_0})\ e^{\frac{-Zr}{2a_0}}$ |
| $R_{n,n-1}\ \propto\ r^{n-1}\ e^{-Zr/na_0}$ | $\mu = \frac{m_1 m_2}{m_1+m_2}$ | $\langle\psi_{n\ell m}\|\frac{e^2}{r}\|\psi_{n\ell m}\rangle = \frac{Ze^2}{n^2 a_0} = \frac{Z\alpha^2\mu c^2}{n^2}$ |

1. What is the maximum wavelength of electromagnetic radiation which can eject electrons from a metal having a work function of 3 eV? (3 points)

2. A nucleus has a radius of 4 Fermis. Use the uncertainty principle to estimate the kinetic energy for a neutron localized inside the nucleus. Do the same for an electron. (3 points)

3. Use the uncertainty principle to estimate the ground state energy of Hydrogen. (4 points)

4. A particle is confined to a box of length $L$ in one dimension. It is initially in the ground state. Suddenly, one wall of the box is moved outward making a new box of length $3L$. What is the probability that the particle is in the ground state of the new box? You may find it useful to know that $\int \sin(Ax)\sin(Bx)dx = \frac{\sin((A-B)x)}{2(A-B)} - \frac{\sin((A+B)x)}{2(A+B)}$ (5 points)

5. A beam of particles of wave-number $k$ (this means $e^{ikx}$) is incident upon a one dimensional potential $V(x) = a\delta(x)$. Calculate the probability to be transmitted. Graph it as a function of $k$. (5 points)

6. A particle of mass $m$ is in a **1 dimensional potential** $V(x)$. Calculate the rate of change of the expected values of $x$ and $p$, ($\frac{d\langle x\rangle}{dt}$ and $\frac{d\langle p\rangle}{dt}$). Your answer will obviously depend on the state of the particle and on the potential. (6 points)

7. Derive the commutators $[L^2, L_+]$ and $[L_z, L_+]$. Now show that $L_+ Y_{\ell m} = C Y_{\ell(m+1)}$, that is, $L_+$ lowers the $L_z$ eigenvalue but does not change the $L^2$ eigenvalue. (8 points)

8. Write the (normalized) state which is an eigenstate of $L^2$ with eigenvalue $\ell(\ell+1)\hbar^2 = 2\hbar^2$ and also an **eigenstate of** $L_x$ with eigenvalue $0\hbar$ in terms of the usual $Y_{\ell m}$. (6 points)

9. Compute the commutators $[A^\dagger, A^n]$ and $[A, e^{iHt}]$ for the 1D harmonic oscillator. (6 points)

10. Assuming $u_n$ represents the $n^{th}$ 1D harmonic oscillator energy eigenstate, calculate $\langle u_n|p|u_m\rangle$. (5 points)

11. Evaluate the "uncertainty" in $x$ for the 1D HO ground state $\sqrt{\langle u_0|(x-\bar{x})^2|u_0\rangle}$ where $\bar{x} = \langle u_0|x|u_0\rangle$. Similarly, evaluate the uncertainty in $p$ for the ground state. What is the product $\Delta p \Delta x$? (10 points)

12. Calculate the Fermi energy for $N$ particles of mass $m$ in a 3D cubic "box" of side $L$. Ignore spin for this problem. (8 points)

13. A Hydrogen atom is in its 4D state ($n = 4, \ell = 2$). The atom decays to a lower state by emitting a photon. Find the possible photon energies that may be observed. Give your answers in eV. (5 points)

14. Using the $\psi_{n\ell m}$ notation, list all the $n = 1, 2, 3$ hydrogen states. (Neglect the esistence of spin.) (5 points)

15. At $t = 0$ a hydrogen atom is in the state $\frac{1}{\sqrt{2}}(\psi_{100} - \psi_{200})$. Calculate the expected value of $r$ as a function of time. (12 points)