Homework I

1. $K_{max} = 3.1 \pm 0.11$ eV for $\lambda = 2000$ Å

$K_{max} = 1.80 \pm 0.07$ eV for $\lambda = 2580$ Å

Photoelectric effect : $\underbrace{\frac{hc}{\lambda}}_{\text{incident photon energy}} - \underbrace{\phi}_{\text{metal work function}} = \underbrace{K_{max}}_{\text{maximum photoelectron energy}}$

So,

$$\left.\begin{aligned} \frac{hc}{\lambda_{2000}} - \phi &= K_{2000} \\ \frac{hc}{\lambda_{2580}} - \phi &= K_{2580} \end{aligned}\right\} \quad K_{2000} - K_{2580} = hc\left(\frac{1}{\lambda_{2000}} - \frac{1}{\lambda_{2580}}\right)$$

$$h = \frac{1}{c}\,\frac{K_{2000} - K_{2580}}{\frac{1}{\lambda_{2000}} - \frac{1}{\lambda_{2580}}} = 6.17 \times 10^{-34}\ \text{J}\cdot\text{s}$$

$$= 3.86 \times 10^{-15}\ \text{eV}\cdot\text{s}$$

$c = 2.998 \times 10^{8}$ m/s

error: $$(\delta h)^2 = \left(\frac{\partial h}{\partial K_{2000}}\,\delta K_{2000}\right)^2 + \left(\frac{\partial h}{\partial K_{2580}}\,\delta K_{2580}\right)^2$$

$$\delta h = \pm 3.87 \times 10^{-16}\ \text{eV}$$

So, $$h = (3.86 \pm 0.39) \times 10^{-15}\ \text{eV}\cdot\text{s}$$

(2) $\lambda = \frac{h}{p}$ (de Broglie wavelength)

(i) 13.6 eV electron

$m_e = 0.511\ MeV/c^2 \longrightarrow$ non-relativistic

$p = \sqrt{2mK}$

So, $\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} \sim \frac{1240\ eV\cdot nm}{\sqrt{2(0.511\times 10^6\ eV)(13.6\ eV)}} \sim 3.33\ \text{Å}$

(ii) 10^5 MeV neutron

$m_n = 939.6\ MeV/c^2 \longrightarrow$ (ultra) relativistic

$\lambda = \frac{hc}{pc} \sim \frac{1240\ eV\cdot nm}{10^{11}\ eV} \sim 1.24\times 10^{-8}\ nm$

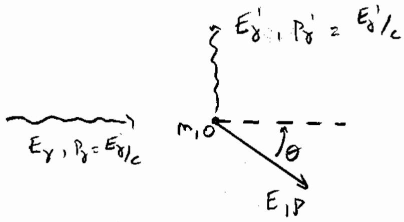
(iii) 1 g, $v = 500$ m/s

$K = \frac{1}{2}mv^2 = \frac{1}{2}(1g)(500\ m/s)^2 = 125\ J \rightarrow$ non-relativistic

$p = mv = (1g)(500\ m/s) = 0.5\ \frac{kg\cdot m}{s}$

$\lambda = \frac{h}{p} = 1.33\times 10^{-33}\ m$

(3)



cons. of mom. $\qquad E_\gamma/c = p\cos\theta \qquad (1)$

$$E_\gamma'/c = p\sin\theta \qquad (2)$$

cons. of energy $\qquad E_\gamma + mc^2 = E_\gamma' + \sqrt{(mc^2)^2 + (pc)^2} \qquad (3)$

Dividing (1) by (2) yields $\qquad E_\gamma' = E_\gamma \tan\theta$.

Also, $\qquad p = \dfrac{E_\gamma}{c}\dfrac{1}{\cos\theta}$.

Substituting into (3) we find $\quad E_\gamma(1-\tan\theta) + mc^2 = \sqrt{(mc^2)^2 + \left(\dfrac{E_\gamma}{\cos\theta}\right)^2}$.

Squaring gives $\quad E_\gamma^2(1-\tan\theta)^2 + \cancel{(mc^2)^2} + 2E_\gamma mc^2(1-\tan\theta) = \cancel{(mc^2)^2} + \left(\dfrac{E_\gamma}{\cos\theta}\right)^2$.

$$E_\gamma^2(1-2\tan\theta + \tan^2\theta) + 2E_\gamma mc^2(1-\tan\theta) = E_\gamma^2\sec^2\theta$$

$$\cancel{E_\gamma^2}(\cancel{\sec^2\theta} - 2\tan\theta) + 2\frac{E_\gamma mc^2}{E_\gamma^2}(1-\tan\theta) = \cancel{E_\gamma^2\sec^2\theta}$$

$$-\cancel{2}\tan\theta + \cancel{2}\left(\frac{mc^2}{E_\gamma}\right)(1-\tan\theta) = 0$$

$$\left(\frac{mc^2}{E_\gamma}\right) = \left(1 + \frac{mc^2}{E_\gamma}\right)\tan\theta$$

$$\tan\theta = \frac{mc^2/E_\gamma}{1+\frac{mc^2}{E_\gamma}} = \frac{mc^2}{mc^2+E_\gamma} = 37.7^\circ$$

So, $\quad E_\gamma' = (150\,\text{keV})\left(\dfrac{mc^2}{mc^2+E_\gamma}\right) = 116\ \text{keV}$

$$E_{e^-} = (E_\gamma - E_\gamma') + mc^2 = 545\ \text{keV}$$

$$K_{e^-} = E - mc^2 = E_\gamma - E_\gamma' = 34\ \text{keV}$$

(4) $$u(\nu,T) = \frac{8\pi h}{c^3} \frac{\nu^3}{\exp(h\nu/kT) - 1}$$

$$W(\lambda,T) = \left|\frac{d\nu}{d\lambda}\right| u(\nu,T)$$

$$c = \lambda\nu \longrightarrow \left|\frac{d\nu}{d\lambda}\right| = \frac{c}{\lambda^2}$$

$$W(\lambda,T) = \frac{\cancel{c}}{\lambda^2} \frac{8\pi h}{c^{\cancel{3}2}} \frac{(c/\lambda)^3}{\exp(hc/\lambda kT) - 1}$$

$$= \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}$$

$$E(\lambda,T) = \frac{c}{4} W(\lambda,T)$$

$$= \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1}$$

(5) $\frac{1}{\lambda} = R_y \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$, $R_y = 1.096 \times 10^7$

$n_1 = 1$:

$$\left.\begin{aligned} \frac{1}{\lambda} &= R_y \left(1 - \frac{1}{2^2}\right) \longrightarrow \lambda = 122 \text{ nm} \\ &= R_y \left(1 - \frac{1}{3^2}\right) \longrightarrow \lambda = 103 \text{ nm} \\ &= R_y \left(1 - \frac{1}{4^2}\right) \longrightarrow \lambda = 97 \text{ nm} \end{aligned}\right\} \text{ultraviolet}$$

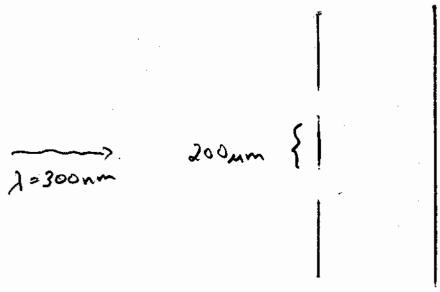
$n_1 = 2$:

$$\left.\begin{aligned} \frac{1}{\lambda} &= R_y \left(\frac{1}{2^2} - \frac{1}{3^2}\right) \longrightarrow \lambda = 657 \text{ nm} \\ &= R_y \left(\frac{1}{2^2} - \frac{1}{4^2}\right) \longrightarrow \lambda = 487 \text{ nm} \\ &= R_y \left(\frac{1}{2^2} - \frac{1}{5^2}\right) \longrightarrow \lambda = 434 \text{ nm} \end{aligned}\right\} \text{visible}$$

$n_1 = 3$:

$$\left.\begin{aligned} \frac{1}{\lambda} &= R_y \left(\frac{1}{3^2} - \frac{1}{4^2}\right) \longrightarrow \lambda = 1.88\ \mu\text{m} \\ &= R_y \left(\frac{1}{3^2} - \frac{1}{5^2}\right) \longrightarrow \lambda = 1.28\ \mu\text{m} \\ &= R_y \left(\frac{1}{3^2} - \frac{1}{6^2}\right) \longrightarrow \lambda = 1.09\ \mu\text{m} \end{aligned}\right\} \text{infrared}$$

(6) $\lambda = 300\ \text{nm}$



$$d \sin\theta = n\lambda$$

$$n = 0 \rightarrow \theta = 0$$

$$n = 1 \rightarrow \theta = \sin^{-1}\left(\frac{\lambda}{d}\right) = 0.09^\circ$$

$$n = 2 \rightarrow \theta = \sin^{-1}\left(\frac{2\lambda}{d}\right) = 0.17^\circ$$

(7) $$\int_{-\infty}^{\infty} f(x)\,\delta(x - x_0)\,dx = f(x_0)$$

$$\psi(x) = \delta(x - x_0)$$

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dx\ \psi(x,0)\, e^{-ipx/\hbar}$$

$$= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dx\ \delta(x - x_0)\, e^{-ipx/\hbar}$$

Consider $f(x) = e^{-ipx/\hbar}$.

Thus, $$\phi(p) = \frac{f(x_0)}{\sqrt{2\pi\hbar}} = \frac{e^{-ipx_0/\hbar}}{\sqrt{2\pi\hbar}}$$

(8) $H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ $\qquad \Delta x \Delta p \sim \frac{\hbar}{2} \rightarrow \Delta x \sim \frac{\hbar}{2\Delta p}$

$H \sim E$

$\rightarrow$ to find minimum, $\frac{\partial H}{\partial p} = 0$

$$\frac{\partial H}{\partial p} = \frac{\partial}{\partial p}\left(\frac{p^2}{2m} + \frac{1}{2}k\left(\frac{\hbar}{2p}\right)^2\right)$$

$$= \frac{p}{m} - \frac{k}{4}\frac{\hbar^2}{p^3} = 0$$

$$\frac{p}{m} = \frac{k}{4}\frac{\hbar^2}{p^3}$$

$$p^4 = \frac{km\hbar^2}{4}$$

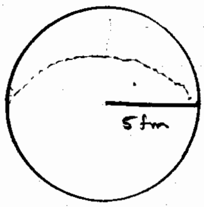
$$p = \left(\frac{km\hbar^2}{4}\right)^{1/4}$$

$$E_{ground} \sim \frac{\sqrt{km\hbar^2}}{2 \cdot 2m} + \frac{1}{2}k\frac{2\hbar^2}{4\sqrt{km\hbar^2}}$$

$$= \frac{1}{4}\sqrt{\frac{k}{m}}\,\hbar + \frac{1}{4}\hbar\sqrt{\frac{k}{m}}$$

$$= \frac{1}{2}\hbar\omega, \qquad \omega = \sqrt{\frac{k}{m}}$$

(9) $r \sim 5 \times 10^{-15}$ fm



$\frac{1}{2}\lambda = 2r$

$\lambda = 4r$

$p = \frac{h}{\lambda} = \frac{h}{4r} \sim 3 \times 10^{-20}$

$= \gamma m v$. (relativistic)

$pc = 9 \times 10^{-12}$ J

$= 62.1$ MeV

→ for an electron,

$$K = \sqrt{(pc)^2 + (mc^2)^2} - (mc^2)$$

$$\sim \sqrt{(pc)^2 + (mc^2)^2}$$

$$\sim pc$$

$$= 62.1 \text{ MeV}$$

→ for a neutron,

$$K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2$$

$$\sim 2.1 \text{ MeV}$$